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THERMAL CONVECTION IN HIGH PRANDTL NUMBER LIQUIDS AT HIGH RAYLEIGH NUMBERS

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Abstract—An experimental investigation was made of the temperature distribution in a heated horizontal layer of high Prandtl number (6 and 18) silicone fluids confined between rigid, parallel, conducting plates. The experiments covered a range of Rayleigh numbers from $7{\cdot}39 \times 10^5$ to $3{\cdot}21 \times 10^8$. The time mean temperature close to the upper and lower boundaries could not be represented by a power law dependence on distance from the boundary. The temperature fluctuations were found to have a characteristic period, as predicted by Chang and Howard. Heat-transfer measurements were in agreement with previous measurements. The distribution of the root mean square temperature fluctuations did not verify any of the proposed theories. The nature of the heat transport processes in the liquid was also studied and the results were compared with other experimental measurements made by Townsend, Croft and Deardorff, and Willis.

NOMENCLATURE

a, wave amplitude (see [1]) [ft];

C, specific heat of experimental fluid (see definition of Rayleigh number) [Btu/lb m deg F]; constant of proportionality in equation (1) [deg F—ft^n]; constant of proportionality in equation (9) [dimensionless];

e, probe thermocouple e.m.f. corresponding to the temperature difference between various stations (z) in the fluid and the reference temperature (T_H) [mV];

$\bar{e}$, time mean probe thermocouple e.m.f. [mV];

e', probe thermocouple e.m.f. fluctuations ($= e - \bar{e}$) [mV];

$\tilde{e}'$, r.m.s. value of the probe thermocouple e.m.f. fluctuations [mV];

g, acceleration of gravity [$4{\cdot}17 \times 10^8$ ft/h^2];

h, coefficient of heat transfer [Btu/h ft^2 °F];

k, thermal conductivity of the fluid [Btu/h ft^2 °F/ft];

L, distance between upper and lower surface of the fluid layer [ft];

n, exponent in equation (1) [dimensionless];

Nu, Nusselt number (hL/k) [dimensionless];

Pr, Prandtl number (ν/α) [dimensionless];

R, ratio of conductive to total heat transport [dimensionless];

Ra, Rayleigh number ($g\beta\rho CL^3\Delta T/\nu k$) [dimensionless];

Ra_δ, conduction layer or wall Rayleigh number [see equation (2)], [dimensionless];

T, temperature at various stations (z) in the fluid [°F];

$\bar{T}$, time mean fluid temperature [°F];

T', fluid temperature fluctuations ($= T - \bar{T}$) [deg F];

$\tilde{T}'$, r.m.s. fluctuations of fluid temperature [deg F];

T_H, temperature of the lower (hot) surface of the fluid layer [°F];

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T_c, temperature of the upper (cold) surface of the fluid layer [°F];
T_M, mean temperature of the fluid layer [$(T_H + T_c)/2$] [°F];
T^*, characteristic time of temperature fluctuations ($10^3 \alpha t^*/L^2$) [dimensionless];
t^*, characteristic time of temperature fluctuations [see equation (2)] [h];
$\tilde{w}'$, r.m.s. value of the vertical (z direction) component (w') of the velocity fluctuations [ft/h];
z, coordinate in vertical direction [ft];
z_α, value of z at which it is assumed that half the heat transport is by conduction ($L/2\ Nu$) [18], [ft];
Δe, e.m.f. corresponding to ΔT [mV];
ΔT, temperature difference between upper and lower surfaces of the fluid ($T_H - T_c$) [deg F];
α, thermal diffusivity of the fluid [ft^2/h];
β, coefficient of volume expansion of the fluid [deg F^{-1}];
ν, kinematic viscosity of the fluid [ft^2/h];
ξ, vertical coordinate ($1{\cdot}89\ z/z_\alpha$) [16], [dimensionless]; vertical coordinate ($z/2\sqrt{\alpha t^*}$) [17], [dimensionless];
ρ, density of fluid [lbm/ft^3].

INTRODUCTION

WHEN heat is first applied at the lower surface of a layer of fluid enclosed at the top and the bottom by rigid, horizontal, conducting boundaries, it is transferred to the upper plate by conduction. With increasing heat input, a point is reached where motion of the fluid commences. The initiation of the motion has been shown to depend on the critical value ($\simeq 1{\cdot}70 \times 10^3$) of a non-dimensional parameter, the Rayleigh number (Ra), which is defined in the Nomenclature.

The motion at Rayleigh numbers slightly greater than critical has a regular cellular or roll pattern, which is often associated with the name of Benard. As the Rayleigh number of the system is increased the regular flow pattern breaks up and eventually (at a Rayleigh number† of about 3×10^4) the flow becomes completely disordered [26, 27]. This disordered fluid motion provides a very suitable means for the study of turbulence. The turbulent fluctuations in wind tunnels and other channels are subject to domination by the effects of the mean flow but the flow in a heated fluid layer is not influenced by effects of this type.

This paper describes an experiment in which the temperatures were measured in a heated fluid layer at high Rayleigh numbers and compared with theoretical predictions.

Theories of free convection at high Rayleigh numbers have been proposed by Malkus [19–21], Chang [1], Kraichnan [18], Howard [16], [17] and Herring [15]. The theories of Howard [16] and Herring [15] are in a certain sense closely related to Malkus' work. For this reason particular attention will be paid in this paper to the studies of Malkus, Kraichnan, Howard [17] and Chang [1].

Malkus and Kraichnan have investigated the temperatures in a region of the fluid where turbulent heat and momentum transfer effects are somewhat larger than viscous effects. For high Rayleigh numbers ($Ra \gtrsim 1{\cdot}0 \times 10^5$) the fluid mean temperature was found to depend on a relation of the form

$$\bar{T}(z) = Cz^{-n}. \qquad (1)$$

The value of the exponent n in equation (1) is shown in Table 1 (note that Kraichnan's

Table 1. Values of exponent n in equation (1)

n	z/z_α	
	Kraichnan [18]	Malkus [20, 21]
1	1 to $3{\cdot}2\sqrt{Pr}$	1 to Nu
$\frac{1}{3}$	$3{\cdot}2(\sqrt{Pr})$ to Nu	—

† The exact value appears to depend on the Prandtl number of the fluid [24, 30].

results in this table apply to a fluid with a Prandtl number greater than 0·1).

Chang [1] and, independently, Howard [17] have proposed a thermal layer model for free convection at high Rayleigh numbers. According to this theory, on the average the temperature fluctuations in the fluid should have a characteristic time (t^*) given by

$$t^* = \frac{L^2}{\pi\alpha}\left(\frac{Ra_\delta}{Ra}\right)^{\frac{2}{3}}. \quad (2)$$

In equation (2) the conduction layer Rayleigh number (Ra_δ) is unknown and no means, other than experiment, were suggested for its evaluation. Howard also obtained an expression for the mean temperature distribution and deduced that

$$Nu = \left(\frac{Ra}{Ra_\delta}\right)^{\frac{1}{3}}. \quad (3)$$

Measurements of the temperature in a fluid layer at high Rayleigh numbers have been made by Thomas and Townsend [28], Deardorff and Willis [5, 6], Elder [11], Rossby [24] and Somerscales and Dropkin [27]. The results of these experiments do not seem to provide a definite support for any of the theories.

A number of investigations [2, 3, 28, 29] have been made of the temperature distribution in air over a single, heated horizontal plate. Comparison of these experiments with the theory is not entirely satisfactory because of the lack of an upper bounding surface. However, the measurements of Townsend [29] are of particular interest because of the care with which the experiment was conducted and the nature and extent of the measurements. Townsend was able to show that equation (1) with $n = 1$ represented the time mean temperature data within experimental accuracy. For the root mean square temperature fluctuations $n = 0{\cdot}6$ was suitable. The coefficient of proportionality in both cases was a function of the Nusselt, Rayleigh and Prandtl numbers.

The investigations reported here were intended to make temperature measurements over a very wide range of Rayleigh numbers using high Prandtl number liquids. In particular it was proposed

(i) To investigate the validity of equation (1)

Table 2. Root mean square temperature fluctuations

$$\frac{\tilde{T}'}{\Delta T} = C\left(\frac{Z}{Z_\alpha}\right)^{-n}$$

Reference	C	n	Ra	Notes
Theoretical investigations				
Kraichnan [18]	0·17	1		
	$0{\cdot}18/\sqrt[3]{Pr}$	$\frac{1}{3}$		
Howard [16]	0·18	—		
Herring [15]	0·18	1		
Howard [17]	0·09	—		$\tilde{T}'/\Delta T)_{max} \simeq C$
Experimental investigations				
Townsend [29]	0·09	0·6		Data of reference 16
Rossby [24]	0·078	0·67	$1{\cdot}16 \times 10^6$	$C = \tilde{T}'/\Delta T_{z/z_\alpha} = 1$
	0·052	0·72	$3{\cdot}15 \times 10^6$	where $z/z_\alpha = 1$
	0·047	0·48	$1{\cdot}0 \times 10^7$	when $z/L = \frac{1}{2} Nu$
Deardorff and Willis [6]	0·11	—	$6{\cdot}3 \times 10^5$	Estimated from
	0·10		$2{\cdot}5 \times 10^6$	Figs. 6, 7 and 8 in [6].
	0·09		$1{\cdot}0 \times 10^7$	
Somerscales and Gazda	0·099	0·6	$7{\cdot}39 \times 10^5$	Average value of C,
			$-3{\cdot}2 \times 10^8$	hot and cold boundaries

and the nature of the exponent (n) for both time mean and root mean square temperature fluctuations.

(ii) To investigate the theories of Chang and Howard [equations (2) and (3)].

(iii) To study the nature of the heat transport processes in the liquid. This requires the determination of the time mean temperature gradient at various stations in the fluid which should be of general interest by itself. Furthermore, as pointed out by Deardorff and Willis [5], this procedure has the advantage that it eliminates the dependence of the data on a particular reference temperature. It is therefore possible to compare measurements made in a confined fluid layer, such as those reported here, with the data obtained from air heated in an open box [2, 29].

Particular emphasis was to be placed on the accuracy of the measurements of both heat transfer and fluid temperatures.

APPARATUS AND INSTRUMENTATION

The apparatus used in these experiments consisted of a rectangular container with copper plates at the top and bottom and insulating side-walls (Fig. 1). The upper plate was supported at its four corners on quartz spacers.† The spacers were ground to length within 0·001 in. There was $\frac{1}{32}$ in. clearance between the upper plate and the side walls. This allowed the spacers to be changed without removing the side walls. The surfaces of the plates in

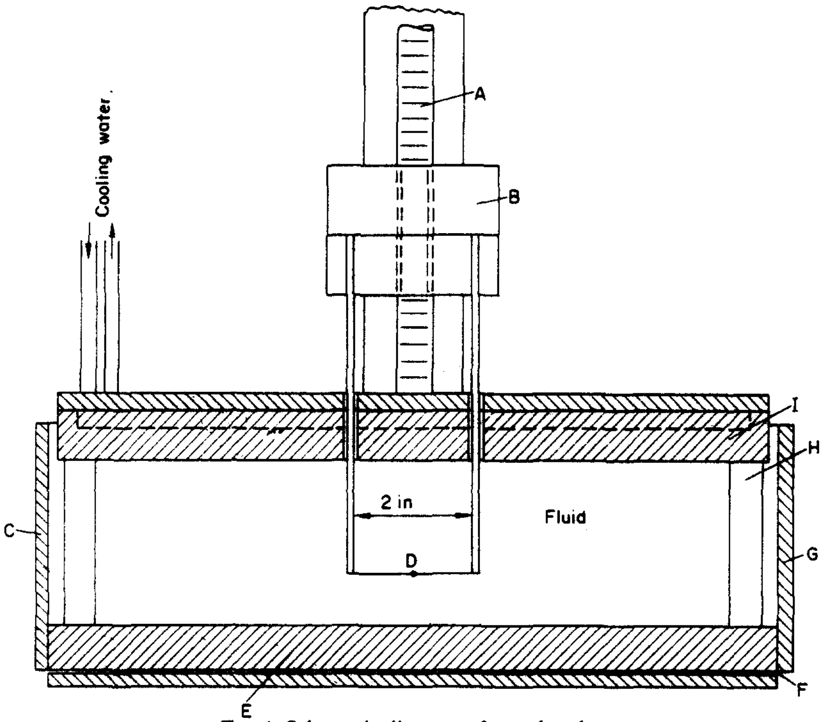

FIG. 1. Schematic diagram of test chamber.
A—Micrometer screw B—Micrometer slide assembly C—Plexiglas side wall D—Thermocouple junction (0·0005 in. dia. wire) E—Hot plate F—Electrical heater G—Plexiglas side wall H—Quartz spacers I—Cold plate.

† The diameters of these spacers were $\frac{3}{8}$ in. (2 in. long), $\frac{5}{16}$ in. (1 in. long), $\frac{1}{4}$ in. ($\frac{1}{2}$ in. long).

contact with the experimental fluid were machined smooth and carefully polished to a mirror finish. The lower plate was made of $7\frac{3}{4}$ in. by $9\frac{3}{4}$ in. by $\frac{1}{2}$ in. thick copper and a $\frac{1}{4}$ in. thick copper back-up plate was attached by screws. Thus the heater plate was about $\frac{3}{4}$ in. thick and sandwiched between its two sections was an electrical heating element.

The heater was made by cutting rectangular strips from a large piece of 32 B & S gage "Nichrome V" sheet, in such a manner that a continuous path was provided for the current (see Fig. 2 of [8]). The heating element was electrically insulated from the copper plates by two 0·010 in. thick sheets of "Micanite". To minimize the effects of electrical noise in the temperature measuring system, the heater was wrapped in grounded shielding and DC power was employed. A precision, regulated power supply was used and this was capable of maintaining a constant output within 0·1 per cent of the setting. The electrical input to the heater was measured by a precision voltmeter (accuracy 1 per cent) and ammeter (accuracy $\frac{1}{2}$ per cent).

The upper surface of the test chamber was cooled by water circulating in channels cut on the top surface of the upper plate. To avoid a non-uniform temperature on the upper plate, the channels were arranged in a spiral so that adjacent channels carry water entering and leaving the system. The rate of heat transfer through the fluid layer can be determined from measurements of the rate of flow and temperature rise of the coolant. The flow rate was measured by a variable area flow meter (accuracy 2 per cent). The temperature of the entering coolant was maintained to within 0·2°F by a constant temperature circulator. The hoses connecting the circulator and the test chamber were carefully insulated. The temperature rise of the coolant was measured by calibrated thermocouples (accuracy 0·1°F) located in a heavy insulating block.

The test chamber was mounted on an insulating support with three leveling screws, which were placed on rubber vibration isolators. The apparatus was surrounded by a 6 in. thick layer of "Styrofoam" (expanded polystyrene).

The temperatures of the upper and lower surfaces were measured by calibrated Chromel–Alumel thermocouples (accuracy 0·1°F), with measuring junctions embedded $\frac{1}{16}$ in. below the liquid–metal interface. These junctions were placed at points distributed over the surfaces so as to indicate the uniformity of the plate temperatures. The reference junction was maintained at 32°F in an ice bath. The e.m.f. output of the thermocouples was read by a Leeds and Northrup K3 potentiometer (accuracy 0·00025 mV).

To measure the liquid temperature in the chamber, a calibrated thermocouple probe (accuracy 0·1°F) was inserted through two 0·035 in. diameter holes in the upper plate. The probe was made from 0·0005 in. dia. Chromel–Alumel wire. Careful examination of the electrically welded junction [13] showed that it was of the same diameter as the wire. The probe was moved vertically by a driving screw with fine pitch and having the minimum of backlash. This screw was attached to the upper plate and carefully aligned in the vertical plane. The location of the probe was determined by a vernier which could be read to 0·001 in. The e.m.f. output of the probe thermocouple was amplified in a low noise (less than 1 n.v.) amplifier (Princeton Applied Research Corporation HR-8 lock-in amplifier†). This amplifier was used in the determination of the time mean temperature (see following section). The amplifier output was recorded on a Sanborn Twin-Viso strip chart recorder to provide a visual record of the temperature fluctuations. The amplifier output was also magnetically recorded using a Lockheed model 411 instrumentation tape recorder. The magnetic recording was

† The amplifier requires the signal to be chopped. A chopper (Stevens–Arnold Inc., type CH1238) with residual noise less than 1 μV was used. For a discussion of the theory of the lock-in amplifier see [22].

used to provide the input to the root mean square computer circuit (see following section). The thermocouple circuit is shown in Fig. 2.

To minimize electromagnetic interference, the thermocouple circuits were completely enclosed in grounded shielding. In this way electrical noise was kept to a level below 1 μV. All unavoidable connections between different metals in the thermocouple circuit were carefully protected from temperature variations by being enclosed in draft free "Styrofoam" enclosures.

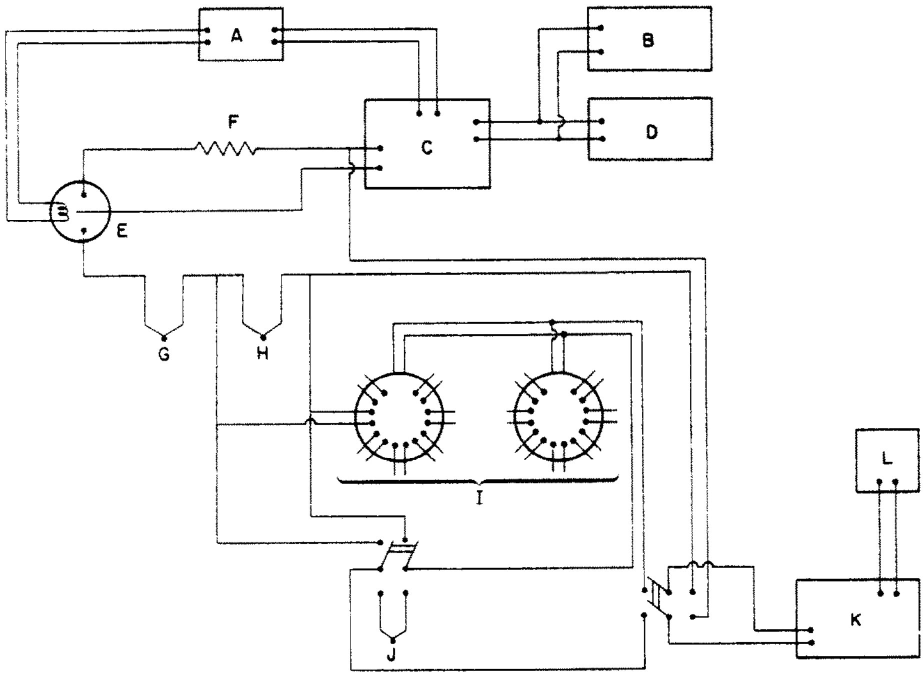

Fig. 2. Thermocouple circuit.
A—Power amplifier B—Recording galvanometer (Sanborn Twin-Viso) C—Lock-in amplifier (Princeton Applied Research HR8) D—Magnetic tape recorder (Lockheed 411) E—Chopper F—Reference resistor (required in lock-in amplification technique) G—Probe junction H—Probe thermocouple reference junction in lower plate I—Plate thermocouple selecter switches J—Plate thermocouple point reference junction K—Potentiometer (Leeds and Northrup K3) L—Electronic null detector.

The thermocouples were calibrated in a specially constructed oil bath [12]. The oil was vigorously circulated and measurements (using the calibrating thermometers) of the temperature distribution at the oil surface showed that the point-to-point variations in temperature were less than 0·1°F. The time constant of the bath was sufficiently long (approximately 5 days) to ensure that the oil temperature did not vary by more than 0·1°F in the time (usually about 1 hr) required to complete the calibration measurements at a given temperature. Standard mercury-in-glass thermometers accurate to 0·1°F were used as the calibrating thermometers.

Dow Corning silicone fluids with a nominal viscosity at 77°F of 0·65 and 2 cSt were used as the experimental fluids. The density and kinematic viscosity of these fluids were measured at room temperature using, respectively, a Cannon–Fenske (capillary) viscometer and a Westphal balance. The variation of density with temperature was calculated from the value of the coefficient of volume expansion supplied by the manufacturer [7]. For the 2 cSt fluid, the temperature dependence of the kinematic viscosity was obtained from the Walther equation [31] and the manufacturer's data [7] on the viscosity–temperature coefficient. It is not possible to use the Walther equation with the 0·65 cSt fluid. For this reason a quadratic relation between kinematic viscosity and temperature was used. When tested on the 2 cSt fluid this equation was found to agree with the Walther equation to within 2 per cent over the temperature range 100–210°F.

EXPERIMENTAL PROCEDURE

Prior to calibration the apparatus was assembled as shown in Fig. 1. The probe was lowered to a point a few thousandths of an inch above the lower plate. The ceramic legs of the probe were adjusted to touch the plate. The set-screws clamping the probe to the driving screw were tightened. The thermocouple tensioning screw was then adjusted until the probe legs were just under tension.

At the end of each run an enlarged plot ($\bar{e}$ vs. z) of the measured mean e.m.f. was made. The plotted distribution was extrapolated to pass through the points $\bar{e} = 0$ and $\bar{e} = \Delta e$. The point (z) at which the curve passed through these points corresponds to $z = 0$ (lower plate) and $z = L$ (upper plate), respectively. In this way the zero position of the thermocouple probe was ascertained. The value of the plate spacing (L) obtained in this way agreed within 0·001 in. with the measured length of the quartz spacers.

The apparatus was filled with the experimental fluid which was degassed by being kept at 165°F for 1 h. Care was taken to ensure that all air bubbles were eliminated from the fluid before measurements were made.

Before each run the electrical power input was set to the chosen level and the system allowed to come to thermal equilibrium, twenty four hours usually sufficed for this.

The e.m.f. (e) corresponding to the temperature difference between various stations in the fluid and the central thermocouple in the lower plate was measured. The probe was held at each station for about 10 min. The time mean e.m.f. ($\bar{e}$) was determined by integration of the probe thermocouple output (e). The integrator formed part of the HR 8 lock-in amplifier mentioned in the preceding section. This amplifier actually incorporates a number of integrators each having a different time constant. A time constant which suppressed the e.m.f. fluctuations, as indicated on the amplifier output meter, to within the desired accuracy of measurement (2·0 μV $\simeq$ 0·1°F) was chosen. The value of the time mean e.m.f. ($\bar{e}$) was then determined by means of the K3 potentiometer (K in Fig. 2), using the HR 8 lock-in amplifier as a null detector. For some of the runs (discussed in a following section) fluctuations with a period of several minutes were observed. In these circumstances the K3 potentiometer setting was adjusted until approximately equal swings of the needle on either side of the null detector (HR8 amplifier) output meter zero were observed. By using a long time constant on the HR8 amplifier, only the low frequency fluctuations caused the output meter needle to oscillate. The potentiometer setting was noted and this procedure was repeated several times. The time mean e.m.f. was obtained from the average of the potentiometer settings.

When the time average of the probe e.m.f. had been determined, the time constant of the integrating circuit was set to a much lower value (0·03 s, which is about ten times as large as the estimated probe time constant). With the potentiometer set to buck-out the mean e.m.f., as determined previously, the output from the low noise amplifier was recorded on the strip chart and the magnetic tape for further analysis.

At the beginning, at the end, and once during each run the temperature of the upper and lower surfaces of the test chamber were measured, the power input, cooling water temperature rise, and room temperature were also noted.

The square of the root mean square value ($\tilde{e}'^2$) of the e.m.f. fluctuations (e') was determined using the analog circuit shown in Fig. 3 [25]. Exponential weighting [9] was used under the assumption that steady state conditions existed before the measurements were made. In the presence of long period fluctuations the measurement period was made long enough for an effective integration of all but the longest period fluctuations. In these circumstances it is possible that the long period fluctuations did not contribute correctly to the computed values of the root mean square temperature fluctuations.† Where the long period fluctuations were absent any small residual drift in the time mean e.m.f. was eliminated by the averaging circuit shown in Fig. 3.

The time mean temperature ($\bar{T} - T_H$) and the root mean square ($\tilde{T}'$) of the temperature fluctuations were obtained from the probe thermocouple calibration table.

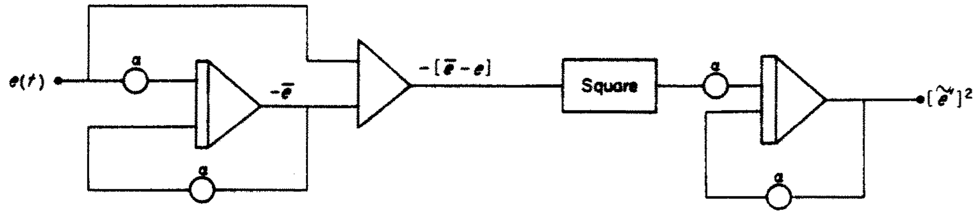

FIG 3. Analog program for obtaining the exponentially mapped past estimate of the variance (α is exponential weighting constant [9, 25]).

Table 3. Experimental conditions

Fluid viscosity† (cSt)	Spacing (L, in.)	Temperature (°F) T_H	T_C	T_M	ΔT	Prandtl number	Rayleigh number	Heat transfer (Btu/h)	Nusselt number
2·0	0·505	94·0	84·6	89·3	9·4	20·2	7.39×10^5	66·1	9·02
2·0	0·505	119·5	100·2	109·9	19·3	16·8	1.76×10^6	166·3	11·1
2·0	1·004	97·0	86·8	91·9	10·2	19·7	6.40×10^6	69·0	17·4
2·0	1·004	115·1	93·8	104·5	21·3	17·6	1.47×10^7	168·6	20·4
2·0	2·013	104·6	93·4	99·0	11·2	18·5	5.97×10^7	63·5	29·3
2·0	2·013	118·7	95·9	107·3	22·8	17·1	1.30×10^8	170·6	38·5
0·65	1·004	110·6	93·6	102·1	17·0	5·64	3.72×10^7	171·3	26·9
0·65	2·013	113·3	95·1	104·2	18·2	5·56	3.21×10^8	171·9	50·8

† Nominal value at 77°F.

† It is also true that exponential weighting does not lead to a valid mean in the presence of long period fluctuations (see further discussion below).

Eight sets of measurements were made and the experimental conditions are summarized in Table 3.

HEAT-TRANSFER MEASUREMENTS

The rate of heat transfer through the fluid layer was determined from measurements of the coolant flow rate and temperature rise, as discussed earlier. Since the estimated heat transfer through the Plexiglas side walls and the quartz spacers was of the same order of magnitude as the estimated heat losses to the surroundings from the cold plate it was not considered necessary to apply any correction to the measured heat transfer. It was not found possible to measure the heat transfer from the temperature gradient in the fluid at the upper and lower boundaries because the dimensions of the conduction layer are so small in the high Prandtl number liquids used in these experiments, that it could not be detected by the measurement techniques employed in the investigation (this is discussed further in a subsequent section). The heat-transfer measurements were found to be in good agreement with previous work [14, 24, 26]. The results are summarized in Fig. 4. A relation of the form

$$Nu = 0{\cdot}196\,Ra^{0{\cdot}283} \quad (4)$$

appears to fit the data within ±5 per cent. This result gives values of the Nusselt number which are about 10 per cent higher than the

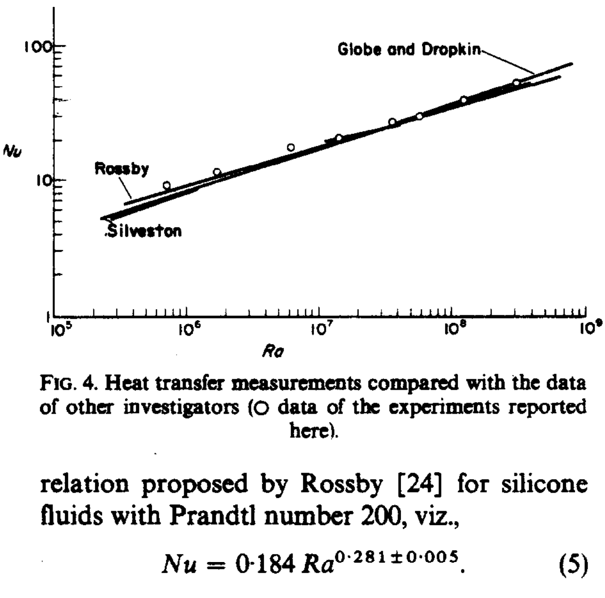

FIG. 4. Heat transfer measurements compared with the data of other investigators (○ data of the experiments reported here).

relation proposed by Rossby [24] for silicone fluids with Prandtl number 200, viz.,

$$Nu = 0{\cdot}184\,Ra^{0{\cdot}281 \pm 0{\cdot}005}. \quad (5)$$

If the Nusselt number is assumed to depend on the cube root of the Rayleigh number, the following equation fits the data within about ±10 per cent

$$Nu = 0{\cdot}085\,Ra^{0{\cdot}333}. \quad (6)$$

Equation (6) is in good agreement with the result proposed by Kraichnan [18] for fluids with Prandtl number greater than 0·1, viz.,

$$Nu = 0{\cdot}089\,Ra^{0{\cdot}333}. \quad (7)$$

However, in view of the better fit to the data provided by equation (4) it is concluded, at least for $Ra < 10^8$, that the appropriate exponent on the Rayleigh number in the heat-transfer relation is less than the value of 0·333 used in equations (6) and (7).

MEAN TEMPERATURE DISTRIBUTION

The plots (Fig. 5) of the mean temperature distribution have a "boundary layer" character with an extensive core of constant temperature fluid. There is a slight asymmetry in these temperature profiles but it is not as pronounced as in the cases reported by Somerscales and Dropkin [27]. The cold boundary layer appears to be a little thinner than the hot boundary layer. In six of the eight runs the temperature in the core fluid is slightly cooler than the mean temperature of the fluid. This is contrary to the observations of Thomas and Townsend [28] and Rossby [24]. No asymmetry of any kind was detected by Deardorff and Willis [6] in their experiments. It is possible that long period fluctuations (with an even longer period than those mentioned before) are present in the fluid. These could produce a very slight and gradual change in the mean temperature of the core during the time required to complete one run.

From Fig. 5 it can be seen that a reversal of the temperature gradient occurs at the edge of the boundary layer for Rayleigh numbers $1{\cdot}47 \times 10^7$ (both hot and cold boundary layers), $1{\cdot}76 \times 10^6$ (hot boundary layer), $6{\cdot}40 \times 10^6$ (cold boundary layer) and $7{\cdot}39 \times 10^5$ (hot boundary layer). These (apart from $Ra = 6{\cdot}40 \times 10^6$) are the Rayleigh numbers at which the long period fluctuations appeared. It seems reasonable to suppose that the two phenomena are related. It was also observed that the long period fluctuations did not have as large a magnitude in the cold boundary layer as in the hot boundary layer.

The amplitudes of the long period fluctuations were approximately proportional to the temperature difference (ΔT) between the upper and

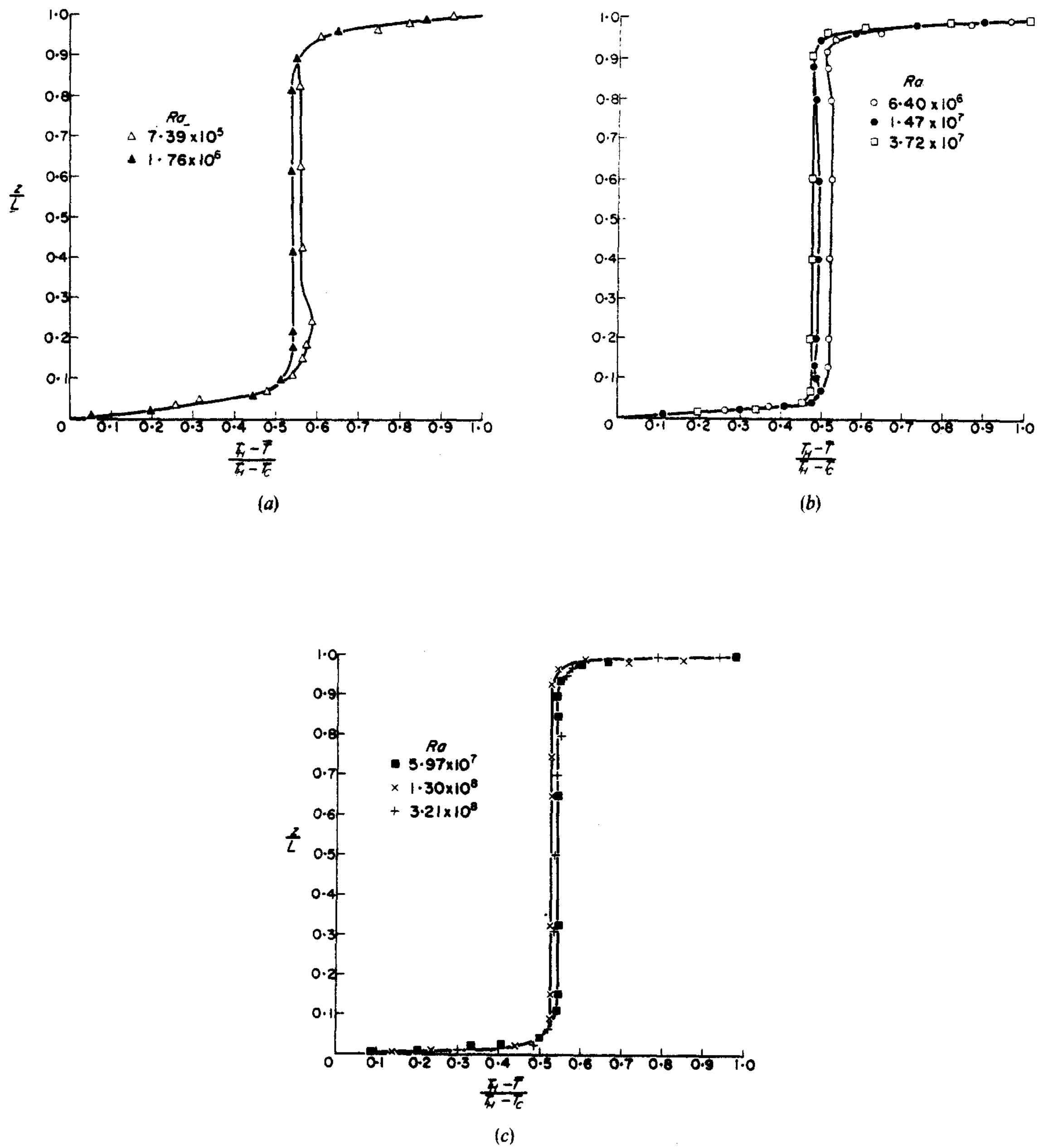

FIG. 5. Mean temperature distribution (some data points omitted for clarity)
(a) $L = 0{\cdot}505$ in.
(b) $L = 1{\cdot}004$ in.
(c) $L = 2{\cdot}013$ in.

lower plates and inversely proportional to the spacing between the plates.

Rossby [24], using a silicone fluid of Prandtl number 100, observed a slight reversal of the temperature gradient in the hot boundary layer at a Rayleigh number of $3{\cdot}15 \times 10^6$. Rossby made no mention of long period fluctuations and it is hard to judge from his records of temperature against time whether they were present or not. A temperature gradient reversal in the cold boundary layer has been reported by Thomas and Townsend [28] in air ($Pr = 0{\cdot}7$) at a Rayleigh number of $6{\cdot}7 \times 10^5$. The records of the temperature fluctuations against time obtained by Thomas and Townsend indicate that long period fluctuations may have been present. However they did not mention their existence. It was suggested that the temperature gradient reversal is due to circulatory motions having dimensions similar to those of the experimental apparatus. A logarithmic temperature distribution (which might be a result of a shear boundary layer caused by circulatory motions) was detected in both the hot and cold boundary layers.

Rossby [24] has made time lapse films of the fluid motions (made visible by the addition of a small quantity of aluminum dust to the liquid). A large scale cellular structure appeared to be present (also observed by Elder [11]) together with a smaller scale "disordered" flow, even at quite high Rayleigh numbers (10^5). Flow of this type has been mentioned by Liepmann in a discussion of [21]. It might be very worthwhile to carry out a program of simultaneous measurement of temperature and velocity together with observations of the flow pattern.

The dimensionless mean temperature was plotted on logarithmic coordinates against Kraichnan's dimensionless distance (z/z_a) for all Rayleigh numbers (see Figs. 6 and 7). The resulting correlation of the data is quite good. Howard's [17] predicted curve for the mean temperature distribution is also plotted in Fig. 6 and it can be seen that the agreement between theory and experiments is satisfactory.

Figures 6 and 7 were examined with a view to detecting regions in which the temperature depended on z^{-1} or $z^{-\frac{1}{3}}$. Lines representing relations of this form have been drawn on the figures. It will further be noted that these

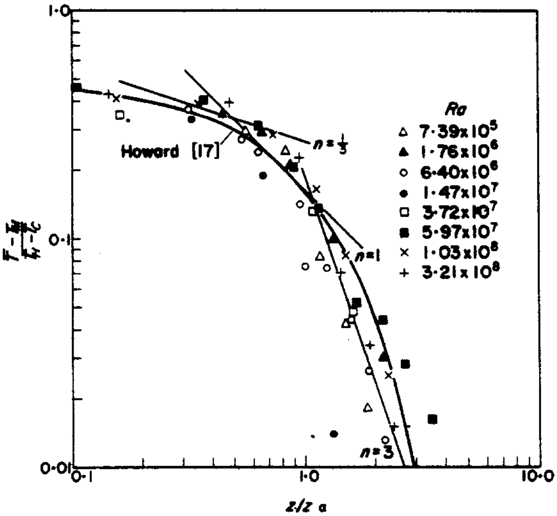

FIG. 6. Correlation of the mean temperature data for the lower plate (some data points omitted for clarity). Note 3·2 ($\sqrt{Pr}$) = 7·85 (Pr = 6), 13·6 (Pr = 18). In this figure distance measured from the lower plate.

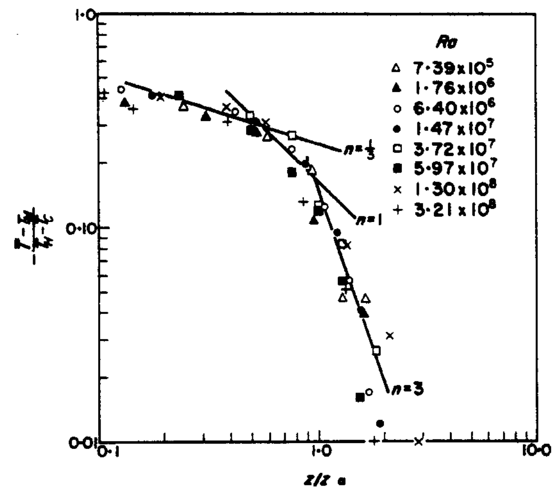

FIG. 7. Correlation of the mean temperature data for the upper plate (some data points omitted for clarity). Note 3·2 ($\sqrt{Pr}$) = 7·85 (Pr = 6), 13·6 (Pr = 18). In this figure distance measured from the upper plate.

lines do not lie in the regions of the flow field proposed by Kraichnan [18] and Malkus [20, 21] (see Table 1). The gradient of the estimated mean curve appears to progress uniformly from zero to about -3 as z increases. It is concluded that a relation of the form of equation (1) cannot predict the mean temperature distribution in high Prandtl number liquids at the Rayleigh numbers encountered in these experiments.

This conclusion is different from the results of Somerscales and Dropkin [27]. However, it is very important when examining data which lie on a curve (as in Figs. 6 and 7) to avoid correlating it with a straight line which is a chord rather than a tangent to the curve. This mistake can be easily made if the ranges of the variables used in plotting the data are too short and if a limited number of data points are plotted. For these reasons the interpretation of the data in [27] is of doubtful validity.†

It is interesting to note that there is an extensive region† for $z/a_\alpha > 1$ in which the temperature varies approximately as z^{-3}. The measurements of Deardorff and Willis [5, 6, 8] in air have a similar characteristic except that the mean temperature dependence is closer to

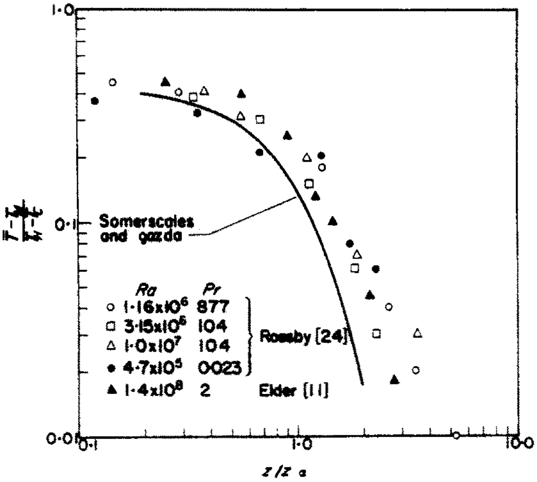

FIG. 8. Mean temperature data (from Fig. 6) compared with the data of other investigators.

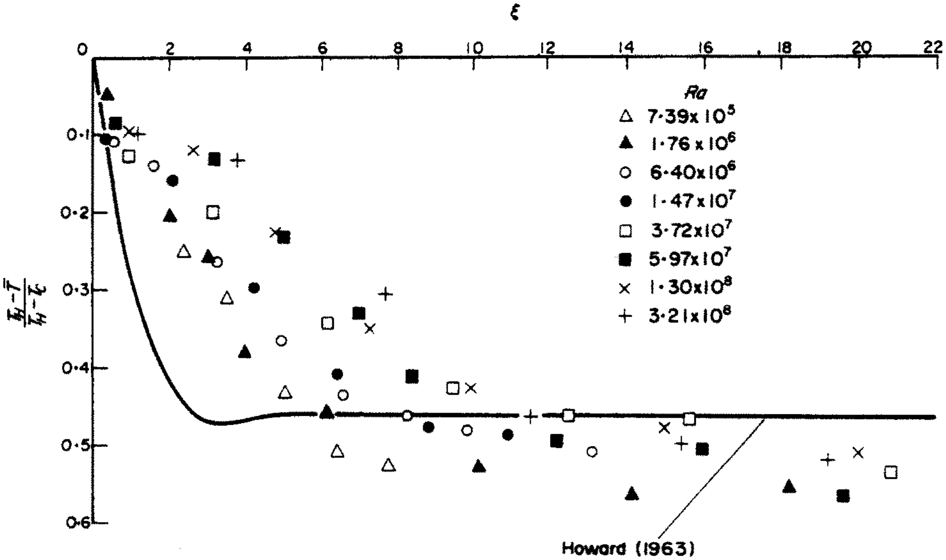

FIG. 9. Howard's [16] correlation of the mean temperature data for the lower plate (distance measured from the lower plate). Some data points are omitted for clarity.

† The values of z_α employed in [27] appear to be higher than those used in the work reported here. The reason for this is not known.

† Elder [11] erroneously stated that the slope of the curve in this region was $-\frac{1}{3}$. The data of Elder is otherwise in good agreement with the results reported here.

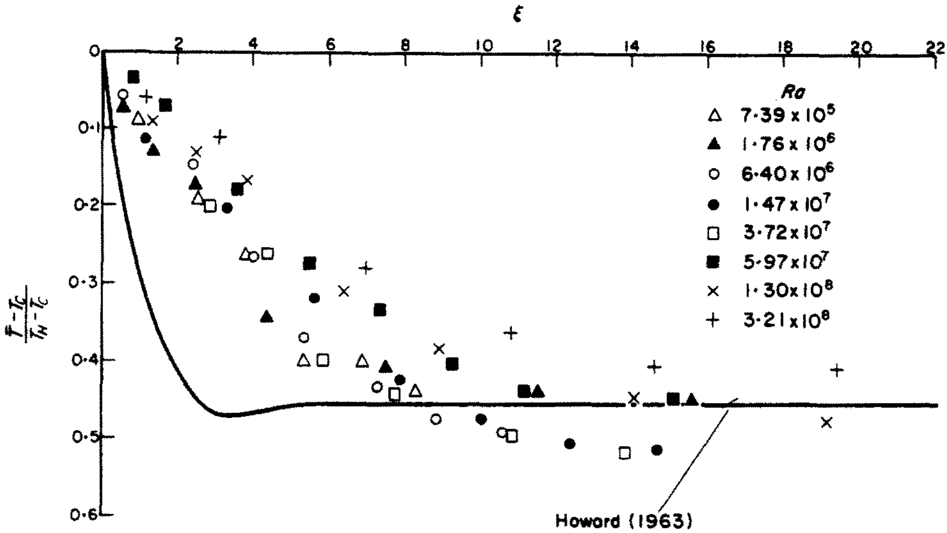

FIG. 10. Howard's [16] correlation of the mean temperature data for the upper plate (distance measured from the upper plate). Some data points are omitted for clarity.

$z^{-1.5}$ (see Fig. 13). This result may reflect the use of spatial averaging by Deardorff and Willis, which they contend is a more efficient procedure for obtaining valid average measurements. Elder [10, 11], whose results are similar to those reported here, used temporal averaging.

The data obtained in the experiments reported here are compared with the results of Rossby [24] and Elder [11] (see Fig. 8). The plotted experimental points in [11] and [24] were replotted and a mean curve drawn by eye through the data.†

Figures 9 and 10 compare the data with Howard's 1963 theory [16]. The predicted temperature has a much steeper slope than the measured values. It can also be seen that the difference between the theory and the measurements decreases as the Rayleigh number decreases. This is in agreement with Howard's calculations which show that the difference between heat transfer measurements and the calculated heat transfer decreases with Rayleigh number.

† It would have been interesting to plot the results of Deardorff and Willis [6] but the graphs in the original reference were found to be too small to allow the extraction of information of reasonable accuracy. For the data of [11] and [24], z_α was calculated from the Globe and Dropkin [14] correlation and from the measured Nusselt number, respectively.

HEAT-TRANSPORT PROCESSES

The heat-transport processes in the liquid layer were examined by determining the gradient of the mean temperature distribution at various stations in the fluid. The gradient was numerically determined from a smoothed curve fitted to the data, at each Rayleigh number, by eye. The ratio (R) of the heat transported by conduction to the total heat transport is plotted against z/z_α on logarithmic coordinates in Figs. 11 and 12. The correlation of the data is quite good, as might be expected from an examination of Figs. 6 and 7.

A mean curve was drawn through the data points by eye and an extrapolation of this curve to values of z/z_α less than 0·1 passes through $R = 1$ at about $z/z_\alpha = 0{\cdot}01$. The maximum value of z_α in the experiments was 0·038 in., so the maximum thickness of the conduction layer is about 0·00038 in. A layer of such dimensions could not be detected by the probe used in these experiments.

In view of this result it was possible to neglect the dimensions of the conduction layer as introduced by Chang into his theory. Then Chang's formula for the heat transfer becomes

$$Nu = 0{\cdot}402\, Ra^{0{\cdot}333} \qquad (8)$$

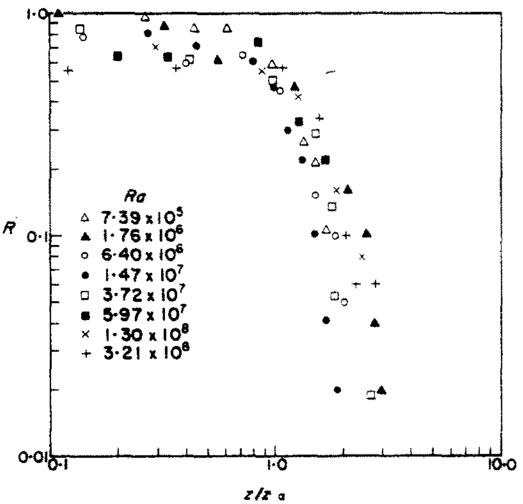

FIG. 11. The ratio (R) of the conductive to the total heat transport for the lower plate (distance measured from the lower plate). Some data points omitted for clarity.

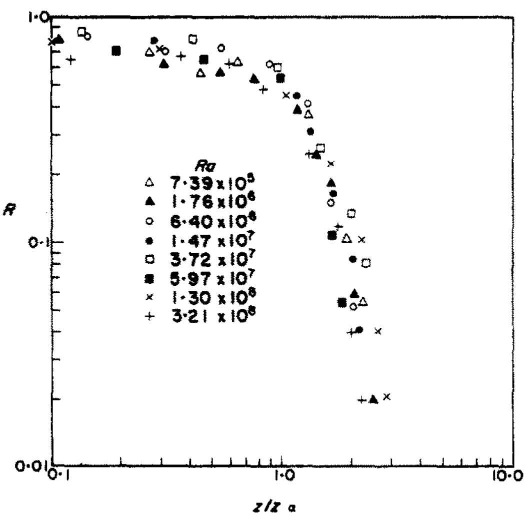

FIG. 12. The ratio (R) of the conductive to the total heat transport for the upper plate (distance measured from the upper plate). Some data points are omitted for clarity.

This result is substantially different from the results [equations (4) and (6)] obtained in the experiments reported here. It is concluded that Chang's wave model is not entirely satisfactory.

The trend of the data for small values of z/z_α is such that the points associated with the higher Rayleigh numbers do not approach $R = 1$ as rapidly as those for small Rayleigh numbers. This suggests, as might be anticipated, that the conduction layer thickness decreases as the Rayleigh number increases.

Earlier, it was pointed out that plotting the gradient of the time mean temperature in the fluid at various stations (z) allows measurements made in confined layers to be compared with measurements made above a single heated plate. This has been done in Fig. 13. The data of Deardorff and Willis [5] for air ($Pr = 0{\cdot}6$) in

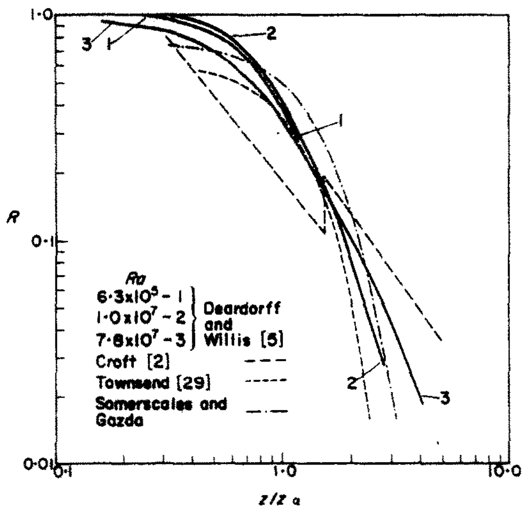

FIG. 13. The data of Fig. 11 compared with the data of other investigators.

a confined fluid layer was taken from the graphs in [5]. In two cases their data attains the value $R = 1$ at some distance from the lower boundary. The lower Prandtl number of air compared to the silcone fluids used here results in a substantially thicker boundary layer which improves the possibility of directly observing a conduction layer (in which R would be equal to unity). In Fig. 13 it can be clearly seen that at large z/z_α the slopes of the curves are somewhat lower than those obtained in the experiments reported here. The results otherwise fall in the same general region of the plot as those obtained here.

and can be represented† by an equation like equation (1) with $n = 0{\cdot}25$ $(0{\cdot}23 \leqslant z/z_\alpha \leqslant 1{\cdot}55)$ and $n = 0{\cdot}50$ $(1{\cdot}55 \leqslant z/z_\alpha \leqslant 6{\cdot}0)$. The correlation of the data using $\bar{T}/\Delta T$ and z/z_α was found to be remarkably good. Figure 13 shows that Croft's results are quite different from the other data which are plotted. Croft made his

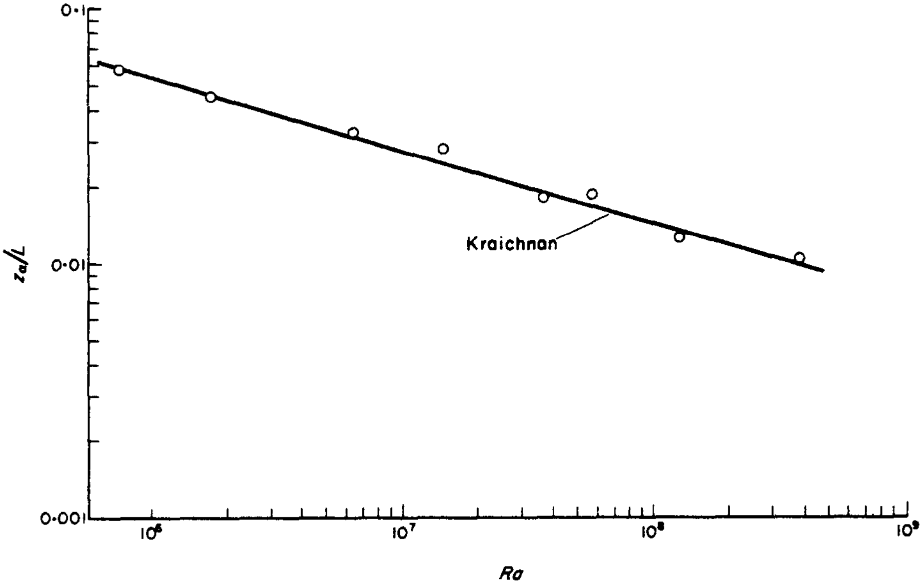

FIG. 14. Kraichnan's [8] prediction of z_α compared with the values obtained from the experiments reported here. (○ data of the experiments reported here).

The data obtained by Townsend [29] in air over a single heated plate is also plotted in Fig. 13. This tends to approach the condition $R = 1$ much more slowly than the results of Deardorff and Willis. Again the data appears to occupy the same region of the plot as other data. The data used in plotting this curve was obtained from a mean curve drawn through the points given by Howard (Fig. 4 in [16]). The gradient was determined by numerical differentiation and z/z_α was calculated from Howard's ξ [16].

The gradient of the time mean temperature measurements made by Croft [2] in air (Pr = 0·7) over a single heated plate is also plotted in Fig. 13. Croft's measurements show that the temperature distribution is split into two regions

temperature measurements using an interferometer whereas all the other data were obtained from measurements made by probes inserted in the fluid. It is possible that the results are a consequence of the method of temperature measurement.

Kraichnan [18] has assumed that the distance (z_α) from the lower surface at which half the heat is transported by conduction is given by

$$z_\alpha = \frac{CL}{2Nu}. \tag{9}$$

Kraichnan arbitrarily chose the coefficient of proportionality (C) to be unity. This result was

† These results were obtained by replotting the data of Croft (Fig. 1 of [2]). z_α was obtained by calculating the heat flux from the dimensionless temperature θ_0 of [2].

compared with the measurements reported here. Mean curves were drawn by eye through the plotted heat transport data for both the hot and the cold boundary layers for each value of the Rayleigh number. A single curve appeared to fit the data from both the hot and cold boundary layers. The value of z at which $R = 0{\cdot}5$ was taken as the experimental value of z_α. This value for z_α is plotted against the Rayleigh number,† Fig. 14. Equation (9) is also plotted in this figure using measured values of the Rayleigh number (essentially, this is equivalent to using equation (4) to obtain the Nusselt number in terms of the Rayleigh number). The agreement between equation (9) and the experimental results is very good.

† The Rayleigh number was used for the abscissa in Fig. 14 rather than the Nusselt number because it allows the data to be plotted without resorting to an exaggerated scale.

TEMPERATURE FLUCTUATIONS

The experimental data were compared with the theories of Howard [17] and Chang [1] by following exactly the same procedure adopted by Rossby [24]. Equation (2) was cast into dimensionless form

$$T^* = \frac{10^3}{\pi}\left(\frac{Ra_\delta}{Ra}\right)^{\frac{2}{3}} \tag{10}$$

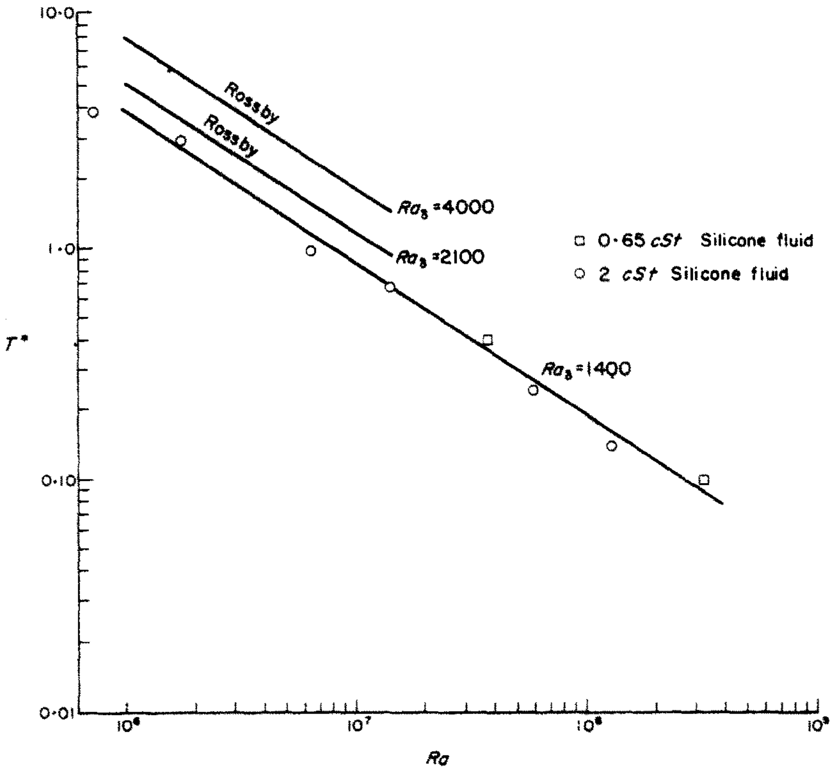

FIG. 15. Characteristic period of the temperature fluctuations compared with Rossby's data [24].

where $T^* = 10^3\alpha t^*/L^2$. T^* was estimated from the record of temperature fluctuations by counting the number of crossings of an approximate mean line in 60 s. This estimate was made at the station where the oscillations appeared to be a maximum. The results are plotted in Fig. 15. The data fits a straight line having the same slope ($-0{\cdot}64$) as the lines drawn by Rossby (Fig. 4.1 of [24]). This is close to the value of $-0{\cdot}66$ proposed by Howard

[17]. Using equation (10) Ra_δ was estimated to be 1400. It is not clear why this value is lower than the values (2100, 4000) proposed by Rossby.

From equation (3) with $Ra_\delta = 1400$ we obtain

$$Nu = 0{\cdot}0893\, Ra^{0.333} \tag{11}$$

This is in quite good agreement with the heat-transfer measurements made in this investigation [equation (6)].

Elder [10] has obtained a wall Rayleigh number (Ra_δ) of about 400 for free convection in a vertical slot filled with water.

The root mean square temperature fluctuations are plotted against z/L in Figs. 16(a)–16(f). The lines shown in these figures were sketched in by eye to assist in "organizing" the data. As mentioned before, caution is advised in interpreting the data from those runs ($Ra = 1{\cdot}47 \times 10^7$, $1{\cdot}76 \times 10^6$ and $7{\cdot}39 \times 10^5$) where long period fluctuations were present. The data characteristically increases from very small values, close to the horizontal boundaries, passes through a maximum and then decreases. The maximum is located at a distance of about $3z_a$ from the horizontal boundaries.

The logarithm of the dimensionless r.m.s. temperature fluctuations is plotted against the logarithm of z/z_a in Figs. 17 and 18. The data is not particularly well correlated by the representation used in these figures.

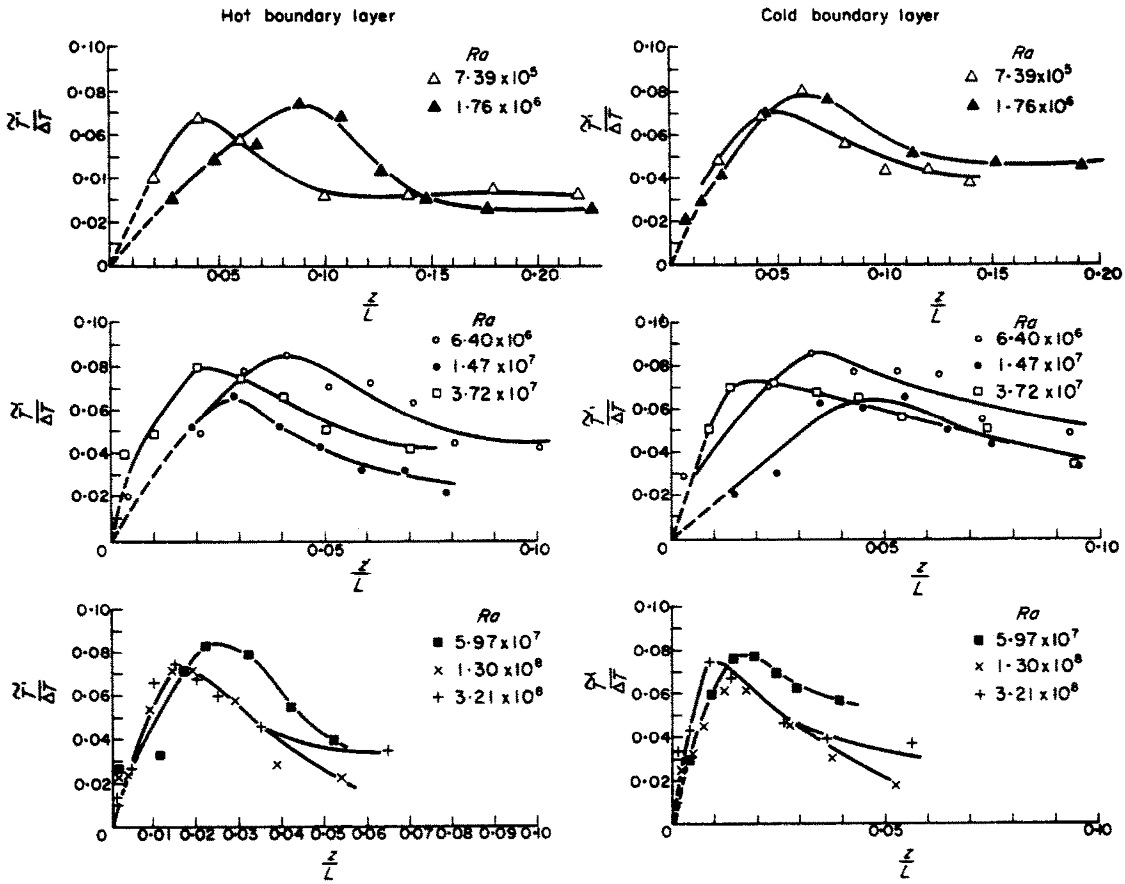

FIG. 16. Dimensionless r.m.s. temperature fluctuations close to the lower and upper surfaces. (a) $L = 0{\cdot}505$ in. (b) $L = 1{\cdot}004$ in. (c) $L = 2{\cdot}013$ in. (d) $L = 0{\cdot}505$ in. (e) $L = 1{\cdot}004$ in. (f) $L = 2{\cdot}013$ in.

Theoretical and experimental relations of the form of equation (1) but applicable to r.m.s. temperature fluctuations are summarized in Table 2. The experimental results tend to have a substantially lower constant of proportionality (C) than predicted by theory. An average value for the exponent (n) as obtained from the measurements, shown in Figs. 17 and 18, appears to be about 0·6. However exponents lying between 0·6 and 0·77 might well fit the data. The value $n = 0{\cdot}6$ was, in fact, only proposed in order to provide a broad comparison with the experiments of Townsend [29]. The fit is generally better in the hot boundary layer. In the case $Ra = 1{\cdot}30 \times 10^8$ an exponent $n = 1$ could be employed. The constant of proportionality was 0·105 for the hot boundary layer and 0·093 for the cold boundary layer, with an average value of 0·099. It is not possible to detect a trend in the value of the exponent as the Rayleigh number increases (cf. Rossby [24]).

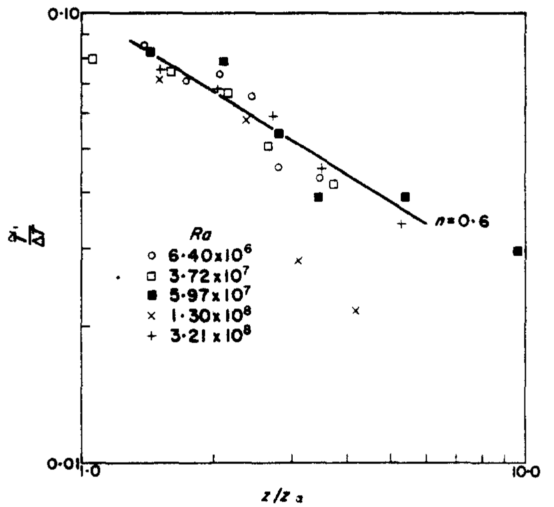

FIG. 17. Correlation of the r.m.s. temperature fluctuations for the lower plate. Distance measured from the lower plate.

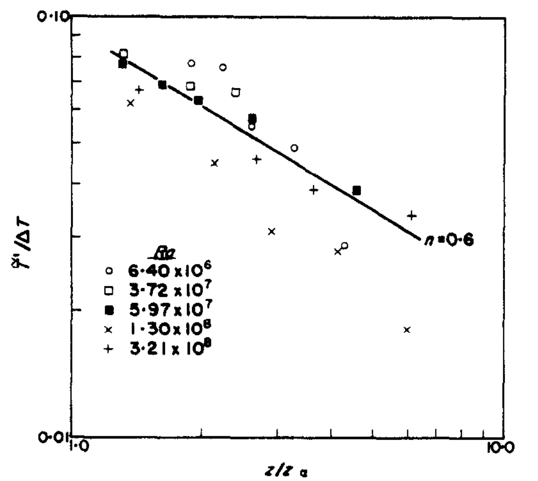

FIG. 18. Correlation of the r.m.s. temperature fluctuations for the upper plate. Distance measured from the upper plate.

ERROR CONSIDERATIONS

The measured values of $\bar{T} - T_H$ are estimated to have an uncertainty of 0·2°F, including estimates of the sampling error. The corresponding estimate for the temperature difference (ΔT) is 0·1 deg F. It was difficult to assign an uncertainty to the values of the dimensionless r.m.s. temperature fluctuations ($\tilde{T}'/\Delta T$) because of the great difficulty in estimating the errors associated with the determination of the r.m.s. temperature fluctuations using the analog computer. Taking this to be ± 10 per cent (conservatively high) it can be said that the dimensionless r.m.s. temperature fluctuations are accurate to within 0·01 at all points (z) in the fluid.†

The height (z) and plate spacing (L) are considered to be accurate to within $\pm 0{\cdot}002$ in. and $\pm 0{\cdot}001$ in., respectively.

The hot and cold plate temperature uniformity was such that the average variation amongst the plate thermocouples was $\pm 0{\cdot}2$°F and $\pm 0{\cdot}4$°F, respectively. During the course of one run (each of which lasted about 4 hr) the temperature difference (ΔT) between the plates did not vary by more than 0·2°F.

The uncertainty in the Nusselt number was

† This also attempts to take into account the effect of the long period fluctuations.

estimated to be ± 7 per cent. The major contribution to this uncertainty is associated with the measurement of the heat flow through the liquid and this uncertainty was about equally divided (± 2 per cent) between the measurement of the flow rate and the measurement of its temperature rise. The Rayleigh number is considered to be accurate to ± 4 per cent.

The experiments were designed so that conditions in the test chamber were free of effects due to the geometry, particularly the aspect ratio, that is, the ratio of horizontal to vertical dimensions. According to the results obtained by Deardorff and Willis [4] there is a possibility that the data for Rayleigh numbers greater than $5{\cdot}97 \times 10^7$, where the aspect ratio was small, was not free from these effects. Although this data appears to have no trends which are markedly different from those of the other data, it must be recognized that the results reported here may only apply for the particular aspect ratio, at which the measurements were made.

Although Deardorff and Willis [4] recommended spatial averaging as a more efficient procedure than temporal averaging, the results obtained in this experiment do not necessarily support the conclusions of Deardorff and Willis. Thus the maximum value of the dimensionless r.m.s. temperature fluctuations ($\bar{T}'/\Delta T$) is 0·08 compared to the value of 0·11 obtained by Deardorff and Willis [6] but this difference, rather than being a consequence of the method of averaging, may arise from the Prandtl number of the experimental fluid. Observations in an earlier series of experiments [27] suggest that for a given Rayleigh number the flow is more disordered at lower Prandtl numbers. Since Deardorff and Willis used air with a nominal Prandtl number of 0·7 in their experiments it would follow that the flow in their experimental system would be more disordered than the flow at the same Rayleigh number in the experiments reported here which used liquids with Prandtl numbers of about 6 and 18. This is, of course, only conjecture and would be a worthy topic for investigation.

CONCLUSIONS

It was not found possible to represent the time mean temperature data by an equation of the form suggested by Kraichnan [18] and Malkus [19], viz.,

$$\bar{T} = Cz^{-n}. \quad (1)$$

Therefore, for high Prandtl number liquids in the range of Rayleigh numbers covered by these experiments ($7{\cdot}39 \times 10^5$–$3{\cdot}21 \times 10^8$) it is concluded that a power law representation of the mean temperature is not applicable. This differs from the results of an earlier series of experiments [27]. However, the methods used for the interpretation of the data in the investigation reported here are considered to be better than those employed in [27].

The data did not fit the predictions of Howard's upper bound theory [16] but this is not to be expected in view of the nature of this theory.

The mean temperature measurements compared satisfactorily with the results of measurements made by Elder [11] and by Rossby [24].

The gradient of the time mean temperature was compared with the results obtained in other experiments [2, 5, 29]. Close to the lower surface the behavior of the curves was quite different but further from the boundary the various data agreed more closely. This is believed to be associated with the Prandtl number of the experimental fluids. The results obtained by Croft [2] were quite different from other measurements. This is probably because Crofts made his temperature measurements with an interferometer whereas the other experimenters used thermocouple or resistance thermometer probes.

The temperature fluctuations were found to have a characteristic "frequency" which varied with the Rayleigh number in accordance with the unsteady conduction theories of Chang [1] and Howard [17]. However, the data did not support Chang's proposed wave model for the fluid motion close to the heated boundary.

The characteristic frequency can be related to the heat transfer in the system. The Nusselt number calculated, according to Howard's procedure [17], from the frequency measurements was in quite good agreement with the heat-transfer measurements made in this investigation. Chang's theory, on the other hand, was found to predict a much higher Nusselt number than the measured values.

The r.m.s. temperature fluctuations could be correlated within ± 30 per cent of the following formula

$$\frac{\dot{T}'}{\Delta T} = 0{\cdot}099\left(\frac{z}{z_{\alpha}}\right)^{-0\cdot 6}. \quad (12)$$

For a Rayleigh number of $1{\cdot}30 \times 10^{8}$ an exponent of -1 seemed to be more appropriate; the same constant of proportionality was applicable.

In the discussion of the data it was pointed out that the measured values of the mean and r.m.s. of the temperature fluctuations for Rayleigh numbers greater than $5{\cdot}97 \times 10^{7}$ may have been affected by the comparatively small ratio of horizontal to vertical distance in the test chamber.

ACKNOWLEDGEMENTS

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P. H. Rothe made a significant contribution to the research by devising and operating an analog circuit for the determination of the r.m.s. temperature fluctuations. This aspect of the research was supported by an Olin Summer Project Grant during the summer of 1967.

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CONVECTION THERMIQUE À DE GRANDS NOMBRES DE RAYLEIGH DANS DES LIQUIDES À NOMBRE DE PRANDTL ÉLEVÉ

Résumé—La distribution de température dans une couche horizontale chauffée d'huiles de silicone à nombre de Prandtl élevé (6 et 18) enfermée entre des plaques conductrices parallèles et regides a été étudiée expérimentalement. Les expériences couvraient une gamme de nombres de Rayleigh de $7,39 \times 10^5$ à $3,21 \times 10^8$.

La moyenne temporelle de la température au voisinage des frontières supérieure et inférieure ne pourrait pas être représentée par une loi en puissance de la distance à l'une de ces frontières. Les fluctuations de température ont une période caractéristique, comme l'avaient prévu Change et Howard. Les mesures du transport de chaleur étaint en accord avec les mesures antérieures. La distribution de la moyenne quadratique des fluctuations de température ne vérifiate aucune des théories proposées. La nature des procesus de transport de chaleur dans le liquide a été aussi étudiée et les résultats ont été comparés avec d'autres mesures experimentales faites par Townsend, Croft et Deardorff et Willis.

Zusammenfassung—Es wurde die Temperaturverteilung in einer gelösten horizontalen Schicht fluider Silicone hoher Prandtlzahl, die durch starre, parallele, leitende Platten begrenzt war, experimentell untersucht. Das Experiment bedeckte einen Bereich der Rayleighzahl von $7,39 \times 10^5$ bis $3,21 \times 10^8$. Die Abhängigkeit der über die Zeit gemittelten Temperatur vom Ort nahe der oberen und unteren Grenze konnte nicht durch ein Potenzgesetz wiedergegeben werden. Die Schwankungen der Temperatur hatten eine charakteristische Periode, wie von Chang und Howard vorausgesagt worden war. Die Messungen des Wärmeübergangs standen in Übereinstimmung mit früheren Messungen. Die Verteilung der mittleren quadratischen Temperaturschwankungen verifizierte keine der vorgeschlagenen Theorien. Die Art des Wärmetransports in der Flüssigkeit wurde ebenfalls untersucht und die Ergebnisse mit Experimenten von Townsend, Croft und Deardorff, und Willis verglichen.

Аннотация—Экспериментально изучено распределение температуры в горизонтальном нагретом слое кремний-органических жидкостей, заключенных между параллельными и проводящими пластинами, при больших числах Прандтля (6 и 18). Опыты проводились в диапазоне чисел Релея от $7,39 \times 10^5$ до $3,21 \times 10^8$. Среднюю температуру во времени вблизи верхней и нижней стенок нельзя представить в виде степенной зависимости от расстояния от них. Найдено, что пульсации температуры имеют характерный период, как и было предсказано Ченгом и Говардом. Измерения по теплообмену находятся в хорошем соответствии с более ранними работами. Полученное в эксперименте среднеквадратичное распределение температурных пульсаций не соответствует ни одной из теорий. Также изучался механизм процесса теплопереноса в жидкости, а результаты сравнивались с экспериментальными данными Таунсенда, Крофта, Диордорффа и Вилье.